Atmospheric Density Estimation Via Satellite Two Line Element Sets

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Abstract
Atmospheric density estimates are a large source of error in low altitude satellite orbit propagations. The density is difficult to model, but by analyzing orbit data from multiple satellites, it can be estimated. To develop better models and improve our current atmospheric density estimates, data is extracted from Space Command’s satellite two line element sets (TLE’s.) With Space Command’s large sample pool of thousands of satellites, analysis of multiple low altitude satellites allows us to develop a model that accurately represents atmospheric density and its changes over time. Errors are reduced by averaging together variations seen for each satellite. Model errors are reduced in the averaging process while true variations in the density remain. This new unique modeling method provides a better estimate of density variations which will result in smaller orbit propagation errors.

1. Introduction
Atmospheric drag is one of the largest causes of perturbations in the orbits of satellites near Earth[^3]. Only the effects of Earth's oblateness are on the same order magnitude for Low Earth Orbits (LEO.) Other perturbations such as third-body effects, solar-radiation, albedo and Earth shine are much smaller, and are often only modeled for satellites with larger orbits.

Effects from Earth's oblateness are easily modeled and parameters for those equations are known to a high degree of accuracy. Drag, however, is much harder to model, mainly due to the uncertainties in the atmospheric density.

Acceleration due to drag is modeled using[^1]:

$$ a_{drag} = -\frac{1}{2} C_D \frac{A}{m} \rho \frac{v_{rel}^2}{v_{ref}^2} $$ (1)

where

- \( a_{drag} \) is the acceleration due to drag,
- \( C_D \) is the coefficient of drag (between 2.0 and 2.1 depending on the satellite),
- \( A \) is the cross-sectional area “normal to the satellite’s velocity vector”,
- \( m \) is the satellite’s mass,
- \( \rho \) is atmospheric density, and
- \( v_{ref} \) is the velocity relative to the Earth’s atmosphere.

It is important to note that atmospheric drag is linearly proportional to atmospheric density. Usually the most difficult parameter to obtain in equation (1) is density. Atmospheric density is very difficult to accurately model because it varies with time and is affected by many factors, the largest coming from solar energy exciting atoms in the upper atmosphere. Complexities that can be accounted for are[^1]:

1. Diurnal Variations - Variations occur daily as the Earth rotates. Maximums occur daily around 2 p.m. local time because of a bulge in the atmosphere caused by solar warming.

2. 27-Day Solar Rotation - The sun rotates every 27 days relative to Earth and has some regions that are more active than others.

3. 11-Year Solar Sun Spot Cycle - More radiation arrives from the sun during the solar maximum.
years when sun spots are more numerous (Figure 1.)

4. Seasonal Variations - The Earth's orbit brings it closer to the sun in January causing small changes in solar radiation received at Earth.

5. Other smaller variations can be caused by, rotating atmosphere, winds, magnetic storms and tides.

Simple, easily implemented density models do not account for any time variations in the atmospheric density. They only change exponentially with altitude. The most popular of these models is the Standard Atmosphere of 1976. This is an “ideal, steady state model of Earth’s Atmosphere at a latitude of 45°N during moderate solar activity” [1,6].

To obtain more accurate estimates of the density, models incorporate inputs that vary with time. The most useful input is the F\textsubscript{10.7} solar flux measurement of radiation from the sun. This is the amount of incoming solar radiation from the sun with a wavelength of 10.7 cm [1].

Models that use F\textsubscript{10.7} as an input include Harris-Priester and Jacchia-Roberts. The Harris-Priester model tabulates densities observed in a solar cycle and interpolation determines approximate densities at certain times. Another commonly accepted model is the Jacchia-Roberts model. The Jacchia-Roberts model “determine[es] exospheric temperature as a function of position, time, solar activity, and geomagnetic activity” [1]. This temperature is used to estimate density from “temperature profiles or from the diffusion equation” [1]. The F\textsubscript{10.7} value along with altitude are both inputs into the Jacchia-Roberts model to determine the atmospheric density.

All these models provide varying degrees of accuracy in estimating atmospheric density; however, the density estimate is still one of the largest errors in the orbit modeling process.

The method presented here uses actual data from satellites to model the density. Density is derived by observing the drag effects on the satellites. This allows for a more accurate estimate of density and can be updated over time to accurately predict the orbit of satellites. Instead of using models from historical measurements, the model is built using recent and real-time measurements that are easily accessible.

2. Procedure

The method uses actual orbit data from US Space Command to estimate past and current changes in the atmospheric density. Satellite two-line elements were retrieved from www.space-track.org [4]. This site allows access to the large database of orbital elements computed for thousands of satellites in orbit around the Earth. These orbital elements are stored in a Two Line Element (TLE) format that includes drag estimates.

The drag estimates are stored in a parameter, B*, which is part of the TLE records [1,2].

\[ B^* = \frac{1}{2} \frac{C_D A}{m} \rho R \]  

where

B* is two-line element drag parameter, and
R is the radius of the Earth, 6378.135 km.

There are many similarities between equation 1 and 2, so knowing B* allows equation 1 to be used without necessarily knowing the individual values of C\textsubscript{D}, A, m, and \( \rho \).

The process to generate the model involves several steps:

1. Download TLE data from Space Command for all the satellites.
2. Extract the B* terms from each TLE.
3. TLE’s can have epoch times at any point in the day. This step uses linear interpolation to estimate the B* terms at the beginning of each day, so that they don’t occur at random times throughout the day.
4. At this point each satellite has roughly the same variation pattern, but there is an unknown scaling factor between each satellite because C\textsubscript{D}, A, and m are different for each satellite. At this step we find the average B* value over the 6 month period and then divide the daily values by that scaling value. This leaves the variations for each satellite centered around the value of 1.
5. Now the models for all the satellites are averaged together to reduce individual measurement errors. This resulting model is the new model that shows how the atmospheric density varies over the six month period.
6. This last step is optional, but the model can now be multiplied by the scaling factors found in step (4) to
get the modeled B* term specific to that satellite. This new model has less noise than the original measurements and fills in any gaps in the data that may have existed in the original TLEs.

This process assumes that the ballistic coefficient, BC, for each satellite remains constant.

\[ BC = \frac{m}{c \rho A} \]  

(3)

Although this assumption is never exactly met, it is a very good approximation for many satellites. It is also assumed that the satellites are in circular orbits and are not maneuvering during the analysis time period.

3. Results

The satellites chosen for this study were in low earth orbit (having greater than 15 revolutions per day, ~570km altitude) and were near circular (having an eccentricity less than 0.01). Data was analyzed for the satellites from 1 Jan to 1 July, 2013.

The B* terms were extracted from the TLEs obtained from Space Command. The format is:

| 1251440 98067A | 04236.5631392 | .000020137 | 00000-0 | 16538=0 | 0 5135 |
| 2 25544 51.6335 341.7760 | 0007976 | 126.2523 | 325.9359 | 15.7046856328903 |

**Figure 2 - TLE Format Example**

Figure 2 shows the TLE format and an example element set for the International Space Station. The highlighted number in line 1 is the B* term that is extracted for this study. The decimal point is assumed, so this example becomes 0.16538x10^{-3}.

**Figure 3 - Raw B* Values for Satellite 38710**

Figure 3 shows the raw B* values for just one of the satellites used in this study.

**Figure 4 - Interpolated B* Values for Satellite 38710**

The figure above is for the same satellite, but the interpolation process has been completed. Gaps have been filled in and B* values are all valid for the beginning of each day.

**Figure 5 - Scaled B* Values for Satellite 38710**

This figure is again for the same satellite, but all the individual values have been divided by the average value. The scaled value has an average of 1 over all of the days.

**Figure 6 - B* Drag Terms**

Figure 6 shows all the data for all the satellites graphed on a single plot. The solid line is the average value for each day. This data shows that satellites have been experiencing similar drag at similar times. The drag appears to go through cycles strongly correlated to the 27 day sun rotation period. The cycles are not perfect and could be affected by irregularities such as sunspots or any of the numerous other reasons listed in the introduction.

From the model, it is also important to note the large changes in maximum and minimums in the cycles. The highest peak is almost a factor of 3 larger than the minimum value. This illustrates the importance of accurately modeling drag or atmospheric density.

This last figure, Figure 7, shows the model scaled back to the original satellite data. The data does not follow along exactly with the raw data points because there is noise associated with each satellite. The averaged B* values for all the satellites represents an idealized situation with less fluctuations occurring due to multiple satellites data being included. The raw data shows the true drag values the satellite experiences, while the scaled model gives an averaged value of multiple satellites. This allows the averaged model to avoid all the large jumps and outliers in points, while still following the variations of the data. Points that are far from the model values can be caused by satellite orbit maneuvering or changes in satellite attitude that in turn affect the coefficient of drag.

4. Application and Improvements

4.1 Computation of Actual Density Values

The model can give good values for B* estimates, but given additional information about a satellite, the density of the atmosphere, \( \rho \) can be calculated.

Rearranging equation (2) gives:

\[
\rho = \frac{2B^*}{R} C_D \rho_0 A
\]

\( B^* \) is obtained from the model, \( R \) is the radius of the Earth and if \( C_D, A \) and \( m \) are known for the satellite, the density value can be computed. It should be noted that this is the average density for satellites at only this altitude above the Earth.

To obtain a full model for all altitudes, several satellites with known properties and various altitudes could be included in this technique. Then using equation (4) individual densities at those altitudes could be determined. To fill in the intermediate altitudes, an exponentially decaying model could be fit from one point to the next.

Another method would be to use a single satellite from this method that has known properties and combine it with an existing model such as the Standard Atmosphere of 1976. In general, changes in the density have a linear nature. If lower altitudes double in density then the upper altitudes do as well.

A scaling correction factor can be obtained for the Standard Atmosphere of 1976 to be used at all altitudes.

\[
\rho_{\text{scale}} = \frac{\rho_{\text{satellite}}}{\rho_{1976}}
\]

where

\( \rho_{\text{scale}} \) is the correction scaling factor,

\( \rho_{\text{satellite}} \) is the actual density from the B* model, and

\( \rho_{1976} \) is the predicted density from the Standard Atmosphere of 1976.

To implement this, the actual density for the known satellite, \( \rho_{\text{satellite}} \), is obtained from the B* model and equation (4). The predicted density from the Standard Atmosphere of 1976, \( \rho_{1976} \), is obtained from that model, and the correction scaling factor, \( \rho_{\text{scale}} \), is computed using equation (5).

From this point on, the Standard Atmosphere of 1976 can be used to calculate densities for all altitudes, but each result is multiplied by \( \rho_{\text{scale}} \) to obtain the corrected
value. For the next point in time, the process is repeated in order to get a new $\rho_{\text{scale}}$.

4.2 Filling in B* Data Gaps and Reducing Noise

Drag data collected from some satellites can have observation gaps and can contain a high level of noise. By using the model obtained, missing B* values can be estimated with reasonable accuracy. Because the model is obtained from an average of many satellites, the noise is also reduced.

Figure 7 demonstrates the filling of gaps and reducing the noise. It is important to note the missing data from the raw data has been filled in; for example, between Julian Dates 2456413.085 and 2456417.358. A four day gap is in this region and the model allows for an approximation throughout that region. Although the averaged B* values do not match the raw data perfectly, it helps show a variation that is realistic for any satellite and gives a reasonable estimate of atmospheric density without the gaps and excess noise.

4.3 Improved Historical Model

By collecting historical data, variations in the atmospheric density can be used to determine drag effects on individual satellites, improving orbit estimates. Because TLE data has been archived for many decades, models could be developed to analyze density changes as far back as the late 1950’s.

This data can also apply to other satellites, because the variations are similar for all the satellites.

4.4 Accurate Short Term B* Predictions

Predicting the B* values from this model can help make estimates of where the satellite will be in the short term. The data won’t be as accurate for future values, so it should only be used in short periods of time, but it allows for calculation of projected drag and can give an estimate of satellites’ locations in the future.

4.5 Long Term B* Estimates

To be able to get a longer term estimate of the location of the satellite, a Fourier analysis could be done on historical data. After identifying the main frequencies of the variations, the model could give a rough average of future values. The analysis would need more data over a longer period of time than analyzed in section 3.

The long term frequencies would have main components for the 11 year sun spot cycle, 27 day solar rotation variations and would probably identify other smaller periodic variations.

4.6 Estimation of Satellite Parameters

In a similar fashion, the ballistic coefficient can be found for satellites. Combining equations (2) and (3) and rearranging:

$$B_C = \frac{\rho R}{2B^*}$$

Again B* is obtained from the model, R is the radius of the Earth and $\rho$ would have to be found via another satellite, or another density model.

4.7 Use In Numerical Integration Models

To predict future locations of satellites, the state of an orbit at an epoch time is often numerically integrated. To account for drag, equation (1) is used in that integration. Because $C_D$, A, m and especially $\rho$ can be hard to determine accurately, the B* model above can be used to determine the combined value, even though the individual components cannot be obtained. From equation (2):

$$\frac{C_D A m}{R} \rho = \frac{2B^*}{R}$$

B* and R can be used to calculate the value of the left hand term. The left hand term can then be used in equation (1) to numerically integrate the satellite’s equation of motion.

4.8 Improvements

Improvements to this research could come from using more satellites over a longer range of time if the Fourier analysis is going to be used. This would allow for a better estimate of the cycles and allow for better predictions in the future.

A source of error in this research is data points that are significantly outside the average cycle. Satellite 36119 has points significantly out of the range that could possibly be thrown out. These points could be due to maneuvers made by the satellite or adjustment of solar panels creating different amounts of drag at those times.

Identifying and using only satellites with “smooth” fluctuations in the B* coefficient would allow for more accurate models. Single outliers that indicate maneuvers could also be removed on a point by point basis.

Because adopting new standard models can be difficult to incorporate into existing software, another area of investigation would be to convert these modeled B* terms
to equivalent $F_{10.7}$ values so that this model could be used as improved inputs for other popular density models such as Harris-Priester and Jacchia-Roberts.

5. Conclusion

Using Space Command TLE’s an accurate model of the atmospheric density variations has been created. This model is an improvement over other traditional models because it uses current actual observations of satellites to estimate the time variations of the atmospheric density. Other popular models don’t model these fluctuations, or they make rough estimates based on observations from many years ago.

The analysis here shows that over a 6 month period the drag effects can increase by as much as 300%, so accurate modeling of the time changing atmospheric density is very important.

Several important applications of this model include historical and future predictions of drag variations, estimates of actual atmospheric density values, a method to estimate satellites’ ballistic coefficients and a method to extract drag values to be used in numerical integration of orbits.

This new model can also be used in many commercial and military applications including; obtaining higher accuracy orbit states, improved satellite collision models, better rocket trajectory estimates and improved re-entry predictions.

Future research could adapt this method as an improved input into other popular models that are widely used to improve accuracy with low amounts of software changes.

6. References


