Estimation of Magnetic Torquing and Momentum Storage Requirements for a Small Satellite

Estimating the external magnetic torquing and internal momentum storage required by a 3-axis attitude controlled small satellite in a circumpolar low Earth orbit.

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Introduction

For a 3 axis attitude controlled satellite to maintain pointing, it must be capable of accounting for environmental torques that impart angular momentum into the satellite. A feasible way of providing this on a small satellite in a circumpolar low earth orbit is through a combination of momentum storage and momentum dumping via magnetic torques. This paper calculates the environmental torques found in low earth orbit, analyzes the process of momentum dumping via magnetic torques for such an orbit, and ultimately estimates the magnetic moment and momentum storage requirements required by the satellite's attitude control system to maintain 3 axis pointing.

I.   Accounting for External Torques

Environmental torques on a satellite in low Earth orbit include the gravity gradient, solar pressure, atmospheric drag, and magnetic torques. Environmental torques are the reason why momentum dumping is necessary; since systematic environmental torques will eventually saturate a satellite's momentum storage facilities, the satellite must be capable of applying external torques to an object outside the satellite system, thus transferring some of the satellite's angular momentum to that external object. Applying such an external torque is referred to as momentum dumping. To dump via magnetic torques, the satellite must control its magnetic moment so that the desired external torque is applied to the earth's magnetic field.

Two ways a satellite can control its magnetic moment include using magnetorquers, or gymballed permanent magnets. For three-axis control of the spacecraft's magnetic moment, the three magnetorquers must be mutually nonparallel, or the permanent magnet must be capable of rotating in three axes (Or equivalently, three permanent magnets must be mounted on rotation-control motors with mutually nonparallel axes).

Where

B is the local magnetic field,

M is the satellite's magnetic moment vector, and

N_m is the corresponding torque vector,

[1]

$$N_m = M \times B$$

(Steyn).

To determine the direction of M, the direction of N_m and B must be known. N_m may be kept through momentum storage knowledge and angular velocity measurements. B may be measured via a 3-axis magnetometer on board the satellite or through knowledge of the spacecraft's position in combination with a model of Earth's magnetic field. In the case of the magnetometer, the satellite's magnetic moment may need to be accounted for in the calculation of the local field from the magnetometer reading. This is especially necessary if permanent magnets are used for attitude control since in the case of magnetorquers they can be turned off briefly during a magnetometer reading.

The environmental torques found in low Earth orbit can be estimated for a given satellite. As an example consider the satellite shown in fig 1.
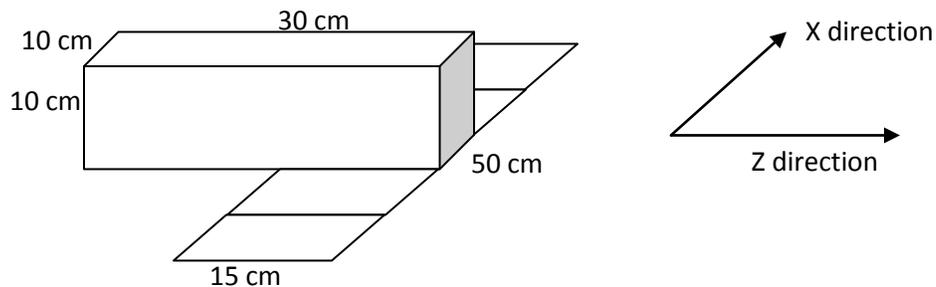

Fig 1; a 30x10x10 cm^3 5kg satellite with off-center lightweight 50x15 cm^2 solar panel.

i. Gravity Gradient

Where

T_g is the maximum gravity torque,

$R \cong 6878000$ m is the orbit radius,

$I_z \cong m * r^2/2 \cong 0.00625$ kg*m^2 is the satellite's moment of inertia in the z-direction,

$I_x \cong m * L^2/12 \cong 0.0375$ kg*m^2 is the satellite's moment of inertia in the x-direction,

$\mu \cong 3.986e(14)$ m^3/s^2 is the gravitational constant for Earth, and

Θ is the maximum deviation of the Z-axis from local vertical,

[2]

$$T_g = \frac{3 * \mu}{2 * R^3} * |I_z - I_y| * \sin(2 * \Theta)$$

(BBB_AD&CS). In the worst case scenario, $T_g \cong 5.74238e(-8)$ N*m.

ii. Solar Radiation

Where

T_{sp} is the solar radiation pressure torque,

$F_s \cong 1367$ W/m^2 is the solar constant,

$C \cong 3 * 10^8$ m/s is the speed of light,

$A_s \cong 0.09$ m^2 is the exposed surface area,

$C_{sp} \cong 0.075$ m is the location of the center of solar pressure,

$C_g \cong 0$ m is the center of gravity location,

$q \cong 1$ (worst case) is the surface reflectance, in the range (0,1), and

i is the angle of incidence to the sun,

[3]

$$T_{sp} = (C_{sp} - C_g) * \frac{F_s}{C} * A_s * (1 + q) * \cos(i)$$

(BBB_AD&CS). In the worst case scenario, $T_{sp} \cong 6.1515e(-8)$ N*m.

iii. Magnetic Field

For a polar orbit, where

T_m is the magnetic torque,

$D \cong 1$A*m^2 is the residual dipole of the satellite (this value is typical),

$M \cong 7.96e(15)$ tesla*m^2 is the magnetic moment of the Earth, and

$R \cong 6878000$ m is the orbit radius,

[4]

$$T_m = D * \frac{2 * M}{R\text{^}3}$$

(BBB_AD&CS). Thus $T_m \cong 4.89279e(-5)$ N*m is the magnetic torque caused by the satellite's residual dipole.

iv. <u>Aerodynamic Disturbance</u>

Where

T_a is the aerodynamic torque,

$\rho \cong 6.98e(-12)$ kg/m^3 is approximately the mean atmospheric density at 500 km altitude,

$C_d \cong 2.25$ is the drag coefficient,

$A \cong 0.09$ m^2 is the exposed surface area,

$V \cong 7613$ m/s is the satellite velocity,

$C_{pa} \cong 0.075$ m is the center of aerodynamic pressure, and

$C_g \cong 0$ m is the center of gravity,

[5]

$$T_a = \left(C_{pa} - C_g\right) * 0.5 * \rho * C_d * A * V\text{^}2$$

(BBB_AD&CS, NRLMSISE). Thus $T_a \cong 3.07202e(-6)$.

In terms of momentum dumping, the worst case scenario is when all of these environmental torques are maximal and in the same direction for an extended period of time. The satellite's momentum dumping facilities must be continuously capable of applying enough torque on the earth's magnetic field for enough time to dump at least as much momentum in the direction of the external torque as is accumulated by the worst-case environmental torque. The magnitude of the worst-case torque is $T_{tot} = T_g + T_{sp} + T_m + T_a \cong 5.21189e(-5)$ N*m.

II.   Approximation of Earth's magnetic field in the satellite reference frame.

To a first approximation, the earth's magnetic field resembles that of a giant bar magnet. Earth's magnetic field points South at the poles and north at the equator. Thus, to a satellite in a circumpolar or near-circumpolar orbit, the earth's magnetic field is constantly changing. As the satellite passes over the North pole, the magnetic field vector points South. As the satellite continues on and passes the equator, the local magnetic field vector rotates around to point North. The local field vector continues rotating to point South as the spacecraft crosses the South pole, and continues to point North again as

the satellite crosses the equator a second time before once again pointing South as the satellite restarts the process at the North pole. Thus an approximation of the Earth's magnetic field direction to the satellite is that of a vector steadily rotating in the satellite's orbital plane.
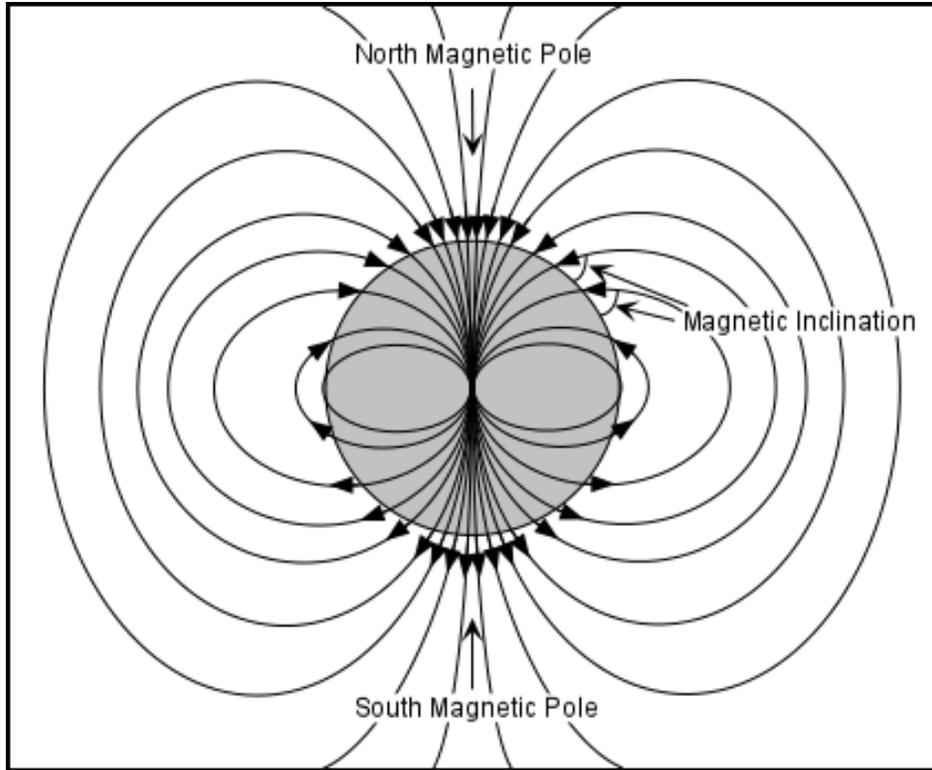

fig 2; the Earth's magnetic field in space. Image obtained from:
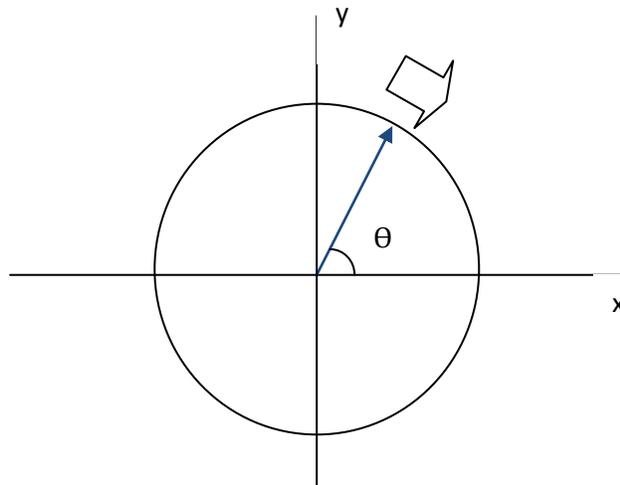http://earthsci.org/education/teacher/basicgeol/platec/platec.html

Fig 3; in the satellite reference frame, the local magnetic field vector (blue) can be approximated by a steadily rotating vector. The circle is in the orbital plane.

5

It is not possible for a satellite to magnetically dump momentum parallel to the local magnetic field. Thus, at any one time, momentum can only be dumped magnetically in two axes of freedom. This is of importance to a satellite meant to sustain three-axis pointing. In order to maintain strict pointing requirements, a satellite which can dump momentum only magnetically must be capable of storing momentum. Also, the torques provided by the momentum dumping device must be some amount greater than the worst case external disturbance torque since magnetic momentum dumping can only be achieved in a direction perpendicular to the direction of the magnetic field surrounding the satellite, and in some cases, the satellite will have to counteract external torques as well as dump stored momentum in order to compensate for not being able to dump momentum in its desired direction all the time. The rest of the paper will estimate the torquing and momentum storage requirements necessary to overcome a worst-case external torque in lieu of these complications. This will involve a 3 dimensional analysis of the interaction of external torques, earth's magnetic field, and the magnetic dipole the spacecraft creates to dump momentum.

III.   Environmental Torque's Worst Case Direction.

We have already discussed that the worst case scenario for an external torque is one with maximum magnitude and constant direction. We have also created our picture of the earth's magnetic field as vector rotating constantly in a plane. In this picture, knowing that external magnetic torques can only be applied perpendicular to the direction of the surrounding magnetic field, what is the worst case scenario for the direction of the external disturbance torque? Suppose that the external disturbance torque was normal to the orbital plane, the plane the earth's magnetic field rotates in. Then this torque is always perpendicular to Earth's magnetic field and thus can be counteracted magnetically without restriction. In fact, for any external disturbance torque vector, the component normal to the orbital plane can be counteracted magnetically without restriction. Thus, momentum dumping is most restricted when the external disturbance torque is in the plane of orbit. Also, we will continue to hold the assumption that an environmental torque with steady direction is worst-case.
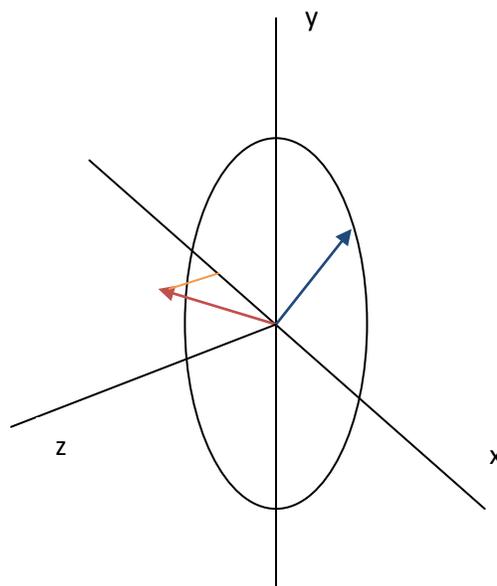
Fig 4. Since the local magnetic field vector (blue) is always in the orbital plane, and since momentum perpendicular to the local magnetic field vector may be dumped directly with no momentum storage required, the component (orange) of the angular momentum to be dumped (red) not in the orbital plane may be dumped at any orientation of the magnetic field.

IV. Calculation of worst case external torquing and momentum storage requirements

Suppose the worst case scenario depicted by figure 5; a worst-case environmental torque in the plane of orbit. Unless the worst-case environmental torque is directly in line with Earth's magnetic field, an external torque can be applied so that some component of it will counteract the environmental torque and thus dump momentum in the desired direction. The smallest magnetic moment the satellite can have and still dump the momentum accumulated by such a worst-case environmental torque is

m, the smallest possible magnetic moment, where

$B \cong 4.89279e(-5)$ tesla is the magnitude of the local magnetic field,

$T \cong 5676$ s is the period of the satellite orbit or equivalently two local magnetic field rotations,

$T_{tot} \cong 5.21189e(-5)$ N*m is the magnitude of the worst-case environmental torque,

$\theta = t * \frac{4\pi}{T}$ is the relationship between time and θ, and

[6]

$$\int_{\theta=0}^{\pi} m * B * \sin(\theta) * \left(\frac{T}{4\pi}\right) * d\theta = \frac{T}{4} * T_{tot}$$

Which implies that

[7]

$$m = \frac{\pi * T_{tot}}{2 * B}$$

And thus that $m \cong 1.67324$ A*m^2 Is the minimum magnetic moment able to overcome the worst-case external torque.
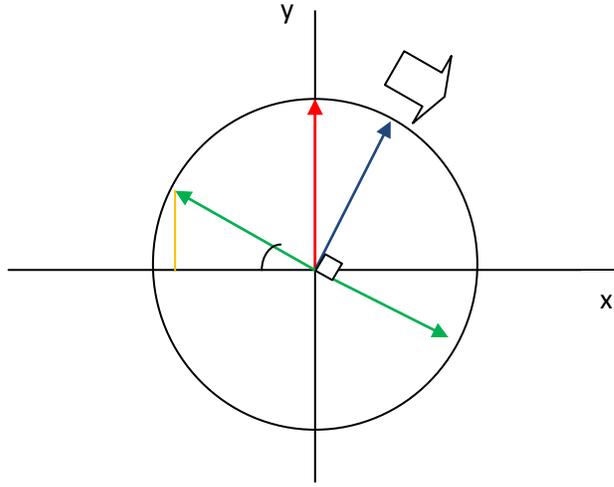
Fig 5; steady external torque and momentum buildup (red), in the plane of rotating local magnetic field (blue). One of the two possible orientations of the satellite's magnetic moment orientations (green) which would dump momentum in the orbital plane have a component in the direction of the environmental torque (orange) causing the momentum buildup (unless red and blue are lined up exactly).

For the smallest magnetic moment case (equations 6 and 7), the momentum storage required in the y-direction as a function of Θ is

[8]

$$M_y(\theta) = \int_{temp=0}^{\theta} (m * B * \sin(temp) - T_{tot}) * \left(\frac{T}{4\pi}\right) * dtemp$$

$$= \frac{-T * (-B * m + \theta * T_{tot} + B * m * \cos(\theta))}{4 * \pi}$$

Figure 6 is the graph of this function over one half-cycle of the momentum vector with the values from equation 6 and 7 plugged in. The corresponding momentum storage required in the x-direction as a function of theta is

[9]

$$M_x(\theta) = \int_{temp=0}^{\theta} m * B * \cos(temp) * \left(\frac{T}{4\pi}\right) * dtemp = \frac{B * m * T * \sin(\theta)}{4 * \pi}$$

Figure 7 is the graph of this function over one half-cycle of the momentum vector.

Being able to predict the external disturbance torque on the satellite could relax the momentum storage requirements. For example, if the satellite knew that it would experience a systematic external disturbance torque for one half-cycle, it could store angular momentum in the opposite direction of the predicted torque before the half-cycle as buffer. If, in this case, the prediction was wrong and the

external disturbance torque was instead in the opposite of the predicted direction, the satellite's momentum storage facilities could become saturated. Assuming no such predictive capabilities, the momentum storage requirement is the difference between the minimum and maximum of the function over the half-cycle.
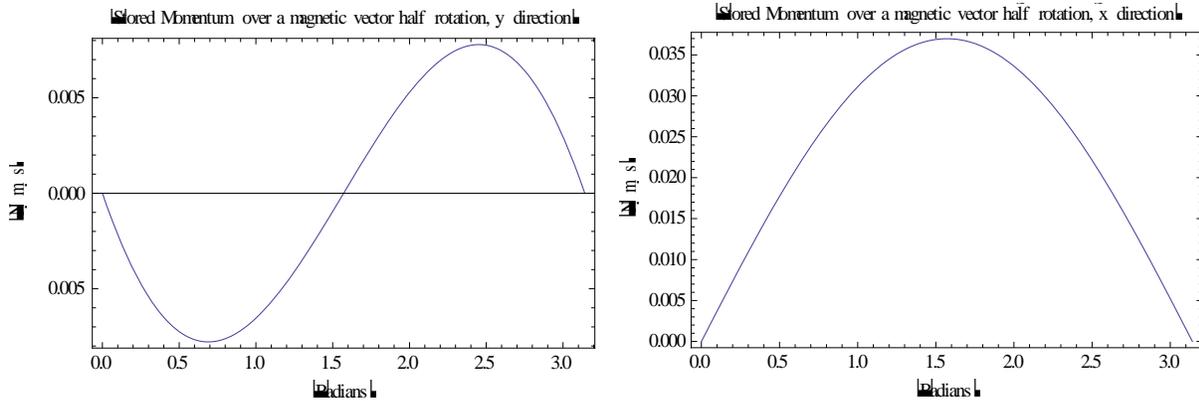

Fig. 6; plots of the momentum storage required to perform the momentum dump using the very minimum magnetic moment. The left plot is for the y-direction, and the right plot is for the momentum stored in the x-direction.

Clearly, for the very minimum magnetic moment momentum dumping strategy, the x-direction requires more momentum storage, since the difference between the maximum and minimum of $M_x(\theta)$ for one half-cycle of θ (magnetic field direction) is greater than the difference is for $M_y(\theta)$. Thus the actual momentum storage requirement in any direction is calculated via the x-direction difference as 0.0369783 N*m*s.

V. Calculation of a more reasonable balance between magnetic moment and momentum storage requirements.

It may be more reasonable to base our magnetic moment and momentum storage requirements off of a momentum dumping scheme that limits its dumping of momentum more than the previously analyzed minimum magnetic moment method. Such a strategy will require higher magnetic moment, but will probably not require as much momentum storage, and possibly also save in other places such as power. The momentum dumping scheme we will analyze next is described in figure 7.
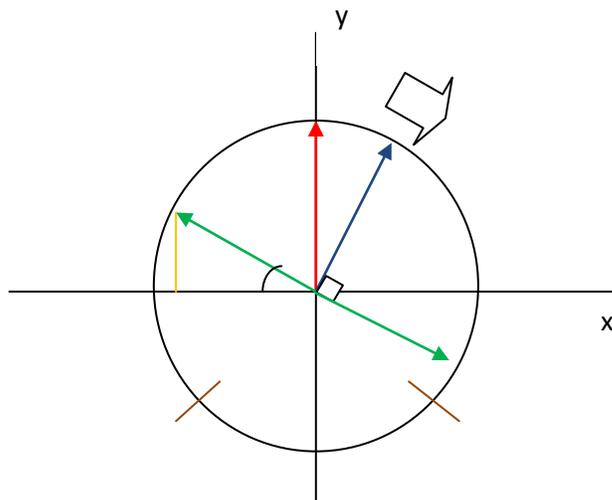
Fig. 7; The two brown hash marks are each 45® from the x-axis. In this momentum dumping scenario we will only dump momentum if one of the green arrows is between the brown hash marks.

The magnetic moment required by this momentum dumping scheme is determined by an equation similar to equation 6,

[10]

$$\int_{\theta=\pi/4}^{3\pi/4} m * B * \sin(\theta) * \left(\frac{T}{4\pi}\right) * d\theta = \frac{T}{4} * T_{tot}$$

Which implies that

[11]

$$m = \frac{\pi * T_{tot}}{\sqrt{2} * B}$$

And thus $m \cong 2.36632$ A*m^2 is the required magnetic moment. Similar to equation 8 and 9, we can calculate the momentum storage required in the y and x directions as

[12]

$$M'_y(\theta) = \int_{temp=0}^{\theta} \frac{T_{tot} * T}{4\pi} dtemp + \int_{temp=\pi/4}^{\theta<3\pi/4} m * B * \sin(temp) * \left(\frac{T}{4\pi}\right) * dtemp$$

[13]

$$M'_x(\theta) = \int_{temp=\pi/4}^{\theta<3\pi/4} m * B * \cos(temp) * \left(\frac{T}{4\pi}\right) * dtemp$$

$\theta < 3\pi/4$ in the limit of integration means the limit is θ, and that the integrand is zero for all $\theta > 3\pi/4$. The evaluation of these functions can be done by mathematica. The maximum of $M'_y(\theta)$ minus the minimum of $M'_y(\theta)$ for θ in [0,π] is 0.0369784 N*m*s. The maximum of $M'_x(\theta)$ minus the minimum of $M'_x(\theta)$ for θ in [0,π] is 0.0153169 N*m*s. Thus this momentum dumping scheme requires the capability to store 0.0369784 N*m*s of angular momentum in any direction.
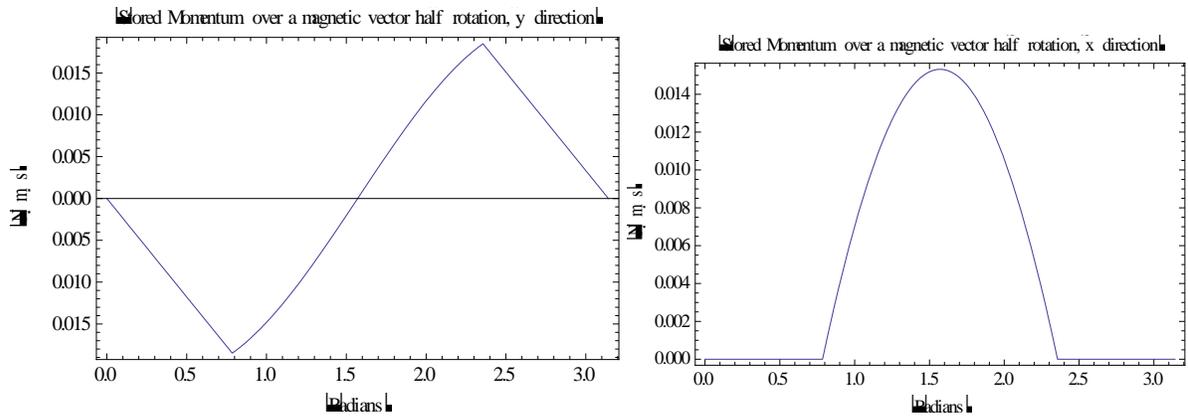
Fig. 8; plots of the momentum storage required to perform the momentum dump using the very minimum magnetic moment. The left plot is for the y-direction, and the right plot is for the momentum stored in the x-direction.

VI. Conclusions

The first momentum dumping scheme minimized the amount of magnetic torque that needed to be applied to counteract environmental torques. That scheme required 1.67324 A*m^2 of magnetic moment providing capability in any direction and 0.0369783 N*m*s of angular momentum storage capability in any direction.

The second momentum dumping scheme attempted to trade some momentum dumping capability for magnetic moment generating capability. It required 2.36632 A*m^2 of magnetic moment capability in any direction and 0.0369784 N*m*s of momentum storage capability in any direction.

In fact, the two methods resulted in similar momentum storage. However, the first method's momentum storage was limited by the x-direction while the second method's momentum storage was limited by the y-direction. Perhaps this means of analyzing momentum storage could find a way to balance the momentum storage required by the two directions and minimize required momentum storage.

References

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2. 'BBB_AD&CS', "Design of an Attitude Determination and Control Subsystem." URL: http://aae.www.ecn.purdue.edu/~crossley/aae490s/BBB_AD&CS.pdf [cited 29 March 2010]