Today:

- Mid Semester Team Evaluations Due now
- Latest Grades posted next Tuesday
  (DD Rev A/B, PDR, and Mid-Semester Team)
- One on One meetings today and tomorrow
- Orbits and Mission Design – Part I
- Launch is in 23 days
Next Class...

Guest Lecture on Attitude Determination and Control (PID Control Law)

Colorado Space Grant Consortium
Next Thursday...

In-Class Team Time

Bring all hardware and be prepared for in-class inspections

Colorado Space Grant Consortium
Tuesday 11/04...

In-Class Mission Simulations

Payloads turned on beginning of class and off at end

Spider
One Minute Report Questions:

- What site or program did you use that gave visual of the orbits?
- Google “orbit tuner”
One Minute Report Questions:

- What type of orbit is the space station in?
  - 51.65 degrees inclination, ~415 km altitude

- Re-boost
One Minute Report Questions:

- How many GEO sats are around Earth?
- How many man-made sats are in space?

- Union of Concerned Scientists…

### Satellite Quick Facts

- Total number of operating satellites: 1084
- LEO: 530
- MEO: 79
- Elliptical: 38
- GEO: 437
- United States: 461
- Russia: 110
- China: 107
- Total number of U.S. Satellites: 461
- Civil: 7
- Commercial: 205
- Government: 115
- Military: 134

*includes launches through 8/31/2013*
One Minute Report Questions:

- How many GEO sats are around Earth?
- How many man-made sats are in space?

- Union of Concerned Scientists…

Satellite Quick Facts (includes launches through 7/31/14)

<table>
<thead>
<tr>
<th>Category</th>
<th>Count</th>
</tr>
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<tbody>
<tr>
<td>Total number of operating satellites:</td>
<td>1,235</td>
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<tr>
<td>United States:</td>
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<td>Russia:</td>
<td>135</td>
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<td>China:</td>
<td>116</td>
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<td>Other:</td>
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<td>MEO:</td>
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<td>GEO:</td>
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<td>Total number of U.S. satellites:</td>
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<td>Government:</td>
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<tr>
<td>Military:</td>
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</tbody>
</table>
One Minute Report Questions:

- Union of Concerned Scientists’ Database and Google Earth Demo…http://www.satellitedebris.net/whatsup/whatsup.htm
Orbit Introduction:

What is an orbit?
- The path of a satellite around the Earth (or any central body)

What shape is it?
- Orbits are conic sections
- Circles, Ellipses, Parabolas, Hyperbolas

How are orbits described?
- Position and Velocity at any one time
- Keplerian Elements (from Kepler’s Laws)
Orbit Introduction:

• What is an orbit?
Orbit Introduction:

• How fast can you throw a baseball?

• Could you throw any of these in to an orbit?  
  - How fast would it have to be going?
Orbit Introduction:

• Let’s figure it out…

\[ v = \sqrt{\frac{GM}{R}} \]

- \( v \) is velocity
- \( G \) is Universal Gravitational Constant
- \( M \) is mass of planet or satellite
- \( R \) is radius of planet or satellite

\[ G = 6.67 \times 10^{-11} \frac{m^3}{(kg \cdot s^2)} \]

\[ M_{Earth} = 5.974 \times 10^{24} \text{ kg} \quad R_{Earth} = 6367000 \text{ m} \]

\[ v_{Earth} = 7910 \ \frac{m}{s} = 17694 \text{ mph} \]
**Orbit Introduction:**

- When in space why do you float? i.e. Weightlessness

\[
\frac{mV^2}{r} = \frac{MmG}{r^2}
\]

\[
V = \sqrt{\frac{MG}{r}}
\]
Kepler and Newton BFFs:

Orbits are conic sections:
- Circle
- Ellipse
- Parabola
- Hyperbola

From Kepler’s Law, the central body is at a focus of the conic section

\[ V = \sqrt{\frac{2MG}{r} - \frac{MG}{a}} \]
Orbit Introduction:

What is an orbit?

- The path of a satellite around the Earth (or any central body)

What shape is it?

- Orbits are conic sections
- Circles, Ellipses, Parabolas, Hyperbolas

How are orbits described?

- Position and Velocity at any one time
- Keplerian Elements (from Kepler’s Laws)
Orbit Definition:

Velocity & Position

- Given position and velocity of a satellite at time \( t \), you can calculate the position and velocity at any other time.
Orbit Definition:

Keplerian Elements

- Semi major axis (a)
- Size
- Eccentricity (e)
- Shape
Orbit Definition:

Keplerian Elements

- Inclination (i)
  - Angle to the Equator
Orbit Definition:
Orbit Definition:

Keplerian Elements

- Right Ascension of Ascending Node (RAAN, \( \Omega \))
- Rotation about the Earth’s Spin Axis
Orbit Definition:

Keplerian Elements

- Argument of Perigee ($\omega$)
- Rotation of the conic section in the plane
**Keplerian Elements**

- **True Anomaly (θ)**
  - Defines the position of a body in orbit
  - Angle between the Position Vector and the vector to Perigee
  - Elliptical only

*Orbit Definition:*
Orbit Definition:
Types of Orbits:

- Geosynchronous/Geostationary (equator)
Types of Orbits:

Geostationary VS. Geosynchronous
Types of Orbits:

- Critical Inclination
Types of Orbits:

- Repeating Ground Trace
Types of Orbits:

- Polar/ Sun Synchronous
Types of Orbits:

- Molniya
Types of Orbits:

- STK Orbit Tuner Demo HERE
Earth, the Moon, Mars, and the Stars Beyond

A Brief Discussion on Mission Design
Universal Gravitation, Applied:

- When in space why do you float? i.e. Weightlessness

\[
mV^2 = \frac{MmG}{r^2}
\]

\[
V = \sqrt{\frac{MG}{r}}
\]
Orbit History:

• 1665 A.D.  
  Isaac Newton
  • At 23, plague while at Cambridge  
  • Went to be one with nature  
  • He studied gravity  
  • Discovered “Newton’s Laws of Motion”  
  • 1666, he understood planetary motion  
  • Did zip for 20 years until Edmund Halley
Newton’s Laws:

1st Law…

\[\text{Body at rest stays at rest, a body in motion stay in motion}\]

2nd Law…

\[F = m \times a\]

3rd Law…

\[\text{For every action, there is an equal and opposite reaction}\]
Newton’s Laws:

Newton Continued...

• 1687, Principia Published
• Law of Universal Gravitation (Attraction)

\[ F = \frac{m_1 m_2 G}{r^2} \]
Newton’s Laws:

Newton Continued...

• 1687, Principia Published
• Law of Universal Gravitation (Attraction)

\[ F = ma = \frac{m_2 V^2}{r} \]

\[ F = \frac{m_1 m_2 G}{r^2} \]
Universal Gravitation, Applied:

• When in space why do you float? i.e. Weightlessness

\[
mV^2 \frac{1}{r} = MmG \frac{1}{r^2}
\]

\[
V = \sqrt{\frac{MG}{r}}
\]
• 1600 A.D.

**Johannes Kepler**

• Used Tycho’s careful Mars observations to smash Aristotle theories
• Presented 3 laws of planetary motion
• Basis of understanding of spacecraft motion
• However, “Why was not understood”
• Calculus?
Kepler’s 3 Laws of Planetary Motion:

1. All planets move in **elliptical orbits**, sun at one focus
Kepler’s 3 Laws of Planetary Motion:

2. A line joining any planet to the sun, sweeps out **equal areas in equal times**
Kepler’s 3 Laws of Planetary Motion:

3. The square of the **period** of any planet about the sun is **proportional** to the cube of the of the planet’s mean **distance** from the sun.

\[ T^2 \propto R^3 \]

\[ \frac{4\pi^2}{T^2} = \frac{GM}{R^3} \]

\[ T = 2\pi \sqrt{\frac{R^3}{\mu}} \]

<table>
<thead>
<tr>
<th>Planet</th>
<th>P (yr)</th>
<th>a (AU)</th>
<th>T^2</th>
<th>R^3</th>
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<tbody>
<tr>
<td>Mercury</td>
<td>0.24</td>
<td>0.39</td>
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<td>Venus</td>
<td>0.62</td>
<td>0.72</td>
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<td>Earth</td>
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<tr>
<td>Mars</td>
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<td>Jupiter</td>
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<td>5.20</td>
<td>142</td>
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<tr>
<td>Saturn</td>
<td>29.5</td>
<td>9.54</td>
<td>870</td>
<td>868</td>
</tr>
</tbody>
</table>

*If you can observe the period of rotation, you can determine the distance*
Orbit History:

Gottfried Leibniz

Isaac Newton
Kepler and Newton BFFs:

Kepler’s Laws...Orbits described by conic sections

Velocity of an orbit described by following equation

\[ v = \sqrt{\frac{2\mu}{r} - \left(\frac{\mu}{a}\right)} \quad \mu = GM \]

For a circle (a=r):

\[ v = \sqrt{\frac{\mu}{r}} \]

For a ellipse (a>0):

\[ v = \sqrt{\frac{2\mu}{r} - \left(\frac{\mu}{a}\right)} \]

For a parabola (a=\infty):

\[ v = \sqrt{\frac{2\mu}{r}} \]
**Orbit Introduction:**

- Let’s figure it out…

\[ v = \sqrt{\frac{GM}{R}} \]

- \( v \) is velocity
- \( G \) is Universal Gravitational Constant
- \( M \) is mass of planet or satellite
- \( R \) is radius of planet or satellite

\[ G = 6.67 \times 10^{-11} \frac{m^3}{(kg \cdot s^2)} \]

\[ M_{Earth} = 5.974 \times 10^{24} \text{ kg} \quad R_{Earth} = 6367000 \text{ m} \]

\[ v_{Earth} = 7910 \frac{m}{s} = 17694 \text{ mph} \]
**Orbit Introduction:**

- **On the moon?**

\[ v = \sqrt{\frac{GM}{R}} \]

- \( v \) is velocity
- \( G \) is Universal Gravitational Constant
- \( M \) is mass of planet or satellite
- \( R \) is radius of planet or satellite

\[ G = 6.67 \times 10^{-11} \frac{m^3}{(kg \cdot s^2)} \]

\[ M_{Earth} = 5.974 \times 10^{24} \text{ kg} \quad R_{Earth} = 6367000 \text{ m} \]

\[ M_{Moon} = 7.350 \times 10^{22} \text{ kg} \quad R_{Moon} = 1738000 \text{ m} \]

\[ v_{Moon} = 1679 \frac{m}{s} = 3756 \text{ mph} \]

\[ v_{Earth} = 7910 \frac{m}{s} = 17694 \text{ mph} \]
Circular Orbit:

For a 250 km circular Earth Orbit

Orbital Velocity

\[
v = \sqrt{\frac{\mu}{r}}
\]

\[
v = \sqrt{\frac{398600.4}{(250 + 6378.14)}}
\]

\[
v = 7.75 \text{ km/sec} = 17,347 \text{ mph}
\]
Circular Orbit:

Orbital Period

\[ P^2 = a^3 \]

\[ P = \frac{\text{circumference}}{velocity} \]

\[ P = 2\pi \sqrt[3]{\frac{r^3}{\mu}} \]

\[ P = 2\pi \sqrt{(250 + 6378.14)^3 \over 398600.4} \]

\[ P = 5,370 \text{ sec} = 89.5 \text{ min} \]
Circular Orbit:

For a 500 km circular Earth Orbit

Orbital Velocity

\[ v = \sqrt{\frac{\mu}{r}} \]

\[ v = \sqrt{\frac{398600.4}{500 + 6378.14}} \]

\[ v = 7.61 \text{ km/sec} = 17,028 \text{ mph} \]
Circular Orbit:

For a 500 km circular Earth Orbit

Orbital Period

\[ P = 2\pi \sqrt{\frac{r^3}{\mu}} \]

\[ P = 2\pi \sqrt{\frac{(500 + 6378.14)^3}{398600.4}} \]

\[ P = 5,676 \text{ sec} = 94.6 \text{ min} \]

Conclusions???
Changing Orbits:

How about 250 km to 500 km

How would you do it?
Changing Orbits:

Changing orbits usually involves an elliptical orbit or Transfer Orbit

Perigee = close
Apogee = far

\[ \Delta v_1 = v_{\text{per}} - v_i \]
\[ \Delta v_2 = v_f - v_{\text{apo}} \]
Changing Orbits:

1) Velocity of initial orbit
   \[ v_i = 7.75 \, \text{km/sec} \]

2) Velocity of final orbit
   \[ v_f = 7.61 \, \text{km/sec} \]

3) Velocity at perigee

4) Velocity at apogee

\[ \Delta v_1 = v_{per} - v_i \]
\[ \Delta v_2 = v_f - v_{apo} \]
Changing Orbits:

Since orbit is elliptical at $V_{per}$ and $V_{apo} a > 0$, so

$$v = \sqrt{\frac{(2\mu)}{r} - \left(\frac{\mu}{a}\right)}$$

where

$$a = \frac{(r_1 + r_2)}{2}$$

$$a = \frac{((250 + 6378.14) + (500 + 6378.14))}{2}$$

$$a = 6753 \ km$$
Changing Orbits:

So back to our $\Delta V$’s

3) Velocity at perigee

$$v_{per} = \sqrt{\frac{(2\mu)}{r}} - \left(\frac{\mu}{a}\right)$$

$$v_{per} = \sqrt{\frac{(2 \times 398600.4)}{(250 + 6378.14)}} - \left(\frac{398600.4}{67.53}\right)$$

$$v_{per} = 7.83 \frac{km}{sec}$$
Changing Orbits:

So back to our ΔV’s

4) Velocity at apogee

\[
v_{\text{per}} = \sqrt{\left(\frac{2\mu}{r}\right) - \left(\frac{\mu}{a}\right)}
\]

\[
v_{\text{per}} = \sqrt{\frac{2 \times 398600.4}{500 + 6378.14} - \frac{398600.4}{6753}}
\]

\[
v_{\text{per}} = 7.54 \frac{\text{km}}{\text{sec}}
\]
Changing Orbits:

1) Velocity of initial orbit
\[ v_i = 7.75 \frac{km}{sec} \]

2) Velocity of final orbit
\[ v_f = 7.61 \frac{km}{sec} \]

3) Velocity at perigee
\[ v_{per} = 7.83 \frac{km}{sec} \]

4) Velocity at apogee
\[ v_{apo} = 7.54 \frac{km}{sec} \]

\[ \Delta v_1 = v_{per} - v_i \]
\[ \Delta v_2 = v_f - v_{apo} \]
Changing Orbits:

Therefore:
\( \Delta V_1 \) is to start transfer

\[
\Delta v_1 = v_{\text{per}} - v_i
\]

\[
\Delta v_1 = 7.83 - 7.75
\]

\[
\Delta v_1 = 0.08 \frac{km}{sec}
\]

\[
\Delta v_1 = 178.9 \text{ mph}
\]
Changing Orbits:

$\Delta V_2$ is to circularize orbit

$\Delta v_2 = v_f - v_{apo}$

$\Delta v_2 = 7.61 - 7.54$

$\Delta v_2 = 0.07 \text{ km/sec}$

$\Delta v_2 = 156.6 \text{ mph}$
Changing Orbits:

What if we did the whole thing in reverse?

Go from 500 to 250 km?

What happens to the answer?

\[
\Delta v_1 = v_{apo} - v_i \\
\Delta v_2 = v_f - v_{per}
\]
Changing Orbits:

1) Velocity of initial orbit
   \[ v_i = 7.61 \text{ km/sec} \]

2) Velocity of final orbit
   \[ v_f = 7.75 \text{ km/sec} \]

3) Velocity at perigee
   \[ v_{per} = 7.83 \text{ km/sec} \]

4) Velocity at apogee
   \[ v_{apo} = 7.54 \text{ km/sec} \]

\[ \Delta v_1 = v_{apo} - v_i \]
\[ \Delta v_2 = v_f - v_{per} \]
Changing Orbits:

Therefore:

$\Delta V_1$ is to start transfer

$\Delta v_1 = v_{apo} - v_i$

$\Delta v_1 = 7.54 - 7.61$

$\Delta v_1 = -0.07 \frac{km}{sec}$

$\Delta v_1 = -156.5 \text{ mph}$
Changing Orbits:

$\Delta V_2$ is to circularize orbit

$\Delta v_2 = v_f - v_{\text{per}}$

$\Delta v_2 = 7.75 - 7.83$

$\Delta v_2 = -0.08 \frac{km}{sec}$

$\Delta v_2 = -178.9 \text{ mph}$
Changing Orbits:

Time to do transfer is the same

\[ P = 2\pi \sqrt{\frac{a^3}{\mu}} \times .5 \]

\[ P = 2\pi \sqrt{\frac{(6753)^3}{398600.4}} \times .5 \]

\[ P = 2,761 \text{ sec} \]

\[ P = 46 \text{ min} \]
How well do you understand Hohmann Transfers?

- 1 to 2?
- 2 to 3?
- 3 to 1?
- 1 to 3?
Circular Orbit:
Changing Orbits:

Also something called “Fast Transfer”

• It is more direct and quicker

• However it takes more fuel

• $\Delta V_1$ and $\Delta V_2$ are much bigger
From Earth Orbit to the Moon:

- Same as changing orbits but....
  - At apogee you don’t have empty space
  - Instead, you have a large and massive object

- Gravity from this object can act as a $\Delta V$ against your spacecraft

- When going to the Moon the following could happen:
  1) Gravity will cause your spacecraft to crash into the surface
  2) Gravity will cause your spacecraft to zip off into space for a long time
Getting to the Moon:

• Gravity Assist

\[ \Delta v_1 \]

\[ \Delta v_2 \]
One Minute Report…

VENUS 1 FLYBY
26 APR 1998

VENUS 2 FLYBY
24 JUN 1999

VENUS TARGETING MANEUVER
3 DEC 1998

LAUNCH
15 OCT 1997

EARTH FLYBY
18 AUG 1999

SATURN ORBIT INSERTION
1 JUL 2004

JUPITER’S ORBIT
11.8 YEARS

29.4 YEARS

Cassini’s speed related to Sun
Moon Video
Apollo XIII:

11. Odyssey splashes down, April 17, 12:07 p.m.
10. Aquarius jettisoned, April 17, 10:43 a.m.
9. Damaged service module jettisoned, April 17, 7:14 a.m.
8. Aquarius fires engine to correct trajectory, April 15, 10:31 p.m.
6. Aquarius fires engine to return to free-return trajectory, April 14, 2:43 a.m.
5. Oxygen tank two in Odyssey explodes, April 15, 9:07 p.m.
4. Odyssey fires engine to leave free-return trajectory, April 12, 7:53 p.m.
3. Odyssey docks with Aquarius, pulling it from third stage, April 11, 3:48 p.m.
2. Saturn V third stage propels Odyssey and Aquarius toward moon, April 11, 1:13 p.m. cst

1. Liftoff, April 11, 1970, 1:13 p.m. cst

7. Aquarius fires engine for PC-2 speed-up burn, April 14, 8:40 p.m.
Apollo XIII:

1. Liftoff, April 11, 1970, 1:13 p.m. CST

2. Saturn 5 third stage propels Odyssey and Aquarius toward moon, April 11, 3:48 p.m.

3. Odyssey docks with Aquarius, pulling it from third stage, April 11, 5:14 p.m.

4. Odyssey fires engine to leave free-return trajectory, April 12, 7:53 p.m.
Apollo XIII:

5. Oxygen tank two in Odyssey explodes, April 13, 9:07 p.m.

6. Aquarius fires engine to return to free-return trajectory, April 14, 2:43 a.m.

7. Aquarius fires engine for PC+2 speed-up burn, April 14, 8:40 p.m.

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Apollo XIII:

1. Liftoff, April 11, 1970, 1:13 p.m. CST

9. Damaged service module jettisoned, April 17, 7:14 a.m.

10. Aquarius jettisoned, April 17, 10:43 a.m.

11. Odyssey splashes down, April 17, 12:07 p.m.
So...

- One switch controls the light bulb
- Light bulb is on 2nd floor
- Can’t see it unless you go upstairs
- Can flip switches as many times
- Can go upstairs once

- Which switch is it?

- If you could go up twice, how would you do it?
- What does a light bulb do?
- Besides light?
- What about heat?
To the Moon for Money:
To the Moon for Money:

Post lunar fly-by
Earth perigee
(42,000 km)
May 16

Trans-lunar injection
May 7

Final geosynchronous
orbit (36,000 km)
late May

(214,000 km)
April 28

(321,000 km)
May 4

5 ¾ days
to moon

3 ¼ days
return to Earth

Moon

Perilune (8000
km) May 13
To the Moon for Money:
To the Moon for Money:
Earth to L1:

Lagrangian Points are orbits about an equilibrium point
There are 5 Lagrangian Points
Earth to L1:

- Solar wind collection in halo orbit about L1 (2 yrs.)
- Return leg (5 mos.)
- Outward leg (3 mos.)
- Lunar orbit
- Parking orbit (optional)
- Positioning for daylight reentry
Earth to Mars:

Initial Orbit  \[ \Delta v_1 \]

Earth Orbit

Mars Orbit

Transfer Orbit  \[ \Delta v_2 \]

Final Orbit
Earth to Beyond:

Say you are in a 250 km orbit...

Orbital Velocity:

\[ v_i = 7.75 \frac{km}{sec} \]

\[ v_i = 17,336 mph \]
Earth to Beyond:

Velocity on parabolic (a=∞) escape trajectory:

\[ v = \sqrt{\frac{(2 \mu)}{r}} \]

\[ v_{esc} = \frac{\sqrt{(2 \times 398600.4)}}{\sqrt{(250 + 6378.14)}} \]

\[ v_{esc} = 10.97 \frac{km}{sec} \]

\[ v_{esc} = 24,539 \text{ mph} \]

\[ \Delta v = 3.22 \frac{km}{sec} \]
Earth to Beyond:

$\Delta V$ needed is...

$\Delta v_{esc} = v_{esc} - v_i$

$= 10.97 - 7.75$

$\Delta v_{esc} = 3.22 \text{ km/sec}$

$\Delta v_{esc} = 7,202 \text{ mph}$
Questions?

Colorado Space Grant Consortium
Orbits:  
*A Brief Historical Look*  

Arthur C. Clarke  

Discovered This Orbit
Ancient Orbit History:

“ORBIT” from Latin word “orbita”
orbitus = circular; orbis = orb

• 1800 B.C.
  Stonehenge
    - Study of the vernal equinox
1500 B.C.: Egyptians and Babylonians

- Written evidence of stellar observations
- Solar Calendar of 365 days
- Time divided into 60 even units
350 B.C.: Greek Thoughts

Aristotle

- Said earth is center of the universe
- Dominated scientific thought for 1800 years
Start of the Heliocentric Model:

1543 A.D.

Nicholas Copernicus
- Said Sun-centered rotations
- Measurements crude but thinking shifts
- Didn’t release findings until the end of his life
Orbit History:

- **1580 A.D.**
  - Tycho Brahe
    - Accurate measurements of planets (Mars) as a function of time
    - Even though telescope had not been invented
Orbit History:

- 1610 A.D.
  Galileo Galilei
  - Good friends with Copernicus
  - Observations with TELESCOPE reinforced
  - Discovered Venus has phases